SUBSIDIARY MATHEMATICS

ADVANCED LEVEL SYLLABUS

NCDC
NATIONAL CURRICULUM DEVELOPMENT CENTRE

REVISED EDITION- 2020

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NCDC
NATIONAL CURRICULUM DEVELOPMENT CENTRE

A product of the National Curriculum Development Centre for the Ministry of Education and Sports with support from the Government of Uganda

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Contents

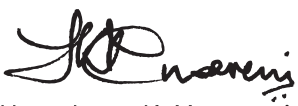
Foreword

The Syllabus of Subsidiary Mathematics has Topics that are largely intended to address the skills in Mathematics that every learner needs in the 21st century. It is incumbent upon the teachers to make the teaching of the topics in this teaching syllabus as practical as possible and relevant to the world of work

Each topic has a background and guidance to the teacher to relate it to its practical application and demystify the areas of emphasis for uniform coverage. This, coupled with the learning outcome, gives the teacher guidance on how to structure the teaching to make it relevant to the learner.

The teaching/ learning strategies suggested in the syllabus is just a guide but is not meant to substitute the rich professional approaches that the teacher may opt to use to deliver knowledge and to develop understanding, skills, values and attitudes.

I therefore endorse this Syllabus as the official document for the teaching and learning of Sub - Maths at the Upper Secondary School level throughout the country.



Hon. Janet K. Museveni

First Lady and Minister of Education and Sports

Acknowledgement

The national Curriculum Development Centre (NCDC) would like to thank everyone who worked tirelessly towards the review of the Uganda Advanced Level Education (UACE) Subsidiary Mathematics curriculum teaching Syllabus

Special thanks go to the Ministry of Education and Sports for supporting the work. NCDC would also like to thank the following for their input: Secondary Schools, Uganda National Examinations Board and Secondary Science and Mathematics Teacher's Programme (SESEMAT). Last but not least, NCDC would like to thank all those who worked behind the scenes to finalise this document.

NCDC takes full responsibility for any errors and omissions in the document and welcomes suggestions to address them.

Grace K. Baguma

Director,

National Curriculum Development Centre

Introduction

Subsidiary Mathematics has been part of the Advanced Level curriculum for a long time but the current Teaching Syllabus was introduced in 2013. The National Curriculum Development Centre has reviewed Subsidiary Mathematics Teaching Syllabus to ease the teaching and learning of the subject. Some content in the previous syllabus has been removed while new content has been added to address learning needs of the 21st Century student. The review has focused on;

i) the mathematics that is applicable in the world of work.

ii) reducing content to make it suitable for all categories of learners.

iii) equipping learners with mathematical skills that are desirable for further learning.

Purpose of the Teaching Syllabus

The Subsidiary Mathematics Teaching Syllabus is meant to:

i) guide teachers handling the subject at Advanced level to teach the learners.

ii) achieve the aims of teaching Subsidiary Mathematics, and standardise the teaching of the subject across the country. It is also meant to guide on how the mathematical skills and competences in this subject can be developed among the learners across the subject topics.

Broad Aims of Education in Uganda

This syllabus contributes towards achieving the broad aims of education listed in the Government White Paper on Education of 1992 as follows;

1. To promote understanding and appreciation of the value of national unity, patriotism and cultural heritage, with due consideration of internal relations and beneficial inter-dependence.
2. To inculcate moral, ethical and spiritual values in the individual and to develop self-discipline, integrity, tolerance and human fellowship.
3. To inculcate a sense of service, duty and leadership for participation in civic, social and national affairs through group activities in educational institutions and the community.
4. To promote scientific, technical and cultural knowledge, skills and attitudes needed to promote development.

5. To eradicate illiteracy and to equip the individual with basic skills and knowledge to exploit the environment for self-development as well as national development, for better health, nutrition and family life and the capacity for continued learning.
6. To contribute to the building of an integrated, self-sustaining and independent national economy.

Aims and Objectives of Secondary Education in Uganda

The Government White Paper on Education provides the following aims and objectives of secondary education in Uganda:

1. Instilling and promoting national unity and an understanding of social and civic responsibilities; strong love and care for others and respect for public property as well as an appreciation of international relations and beneficial international co-operation.
2. Promoting an appreciation and understanding of the cultural heritage of Uganda including languages.
3. Imparting and promoting a sense of self-discipline, ethical and spiritual values, personal responsibility and initiative.
4. Enabling individuals to acquire and develop knowledge and an understanding of emerging needs of society and the economy.
5. Providing up-to-date and comprehensive knowledge in theoretical and practical aspects of innovative production, modern management methods in the field of commerce and industry and their application in the context of social economic development of Uganda.
6. Enabling individuals to develop basic scientific, technological, agricultural, and commercial skills required for self-development.
7. Enabling individuals to develop personal skills of problem-solving, information-gathering and interpretation, independent reading and writing, self-improvement through learning and development of social, physical and leadership skills such as are obtained through games, sports, societies and clubs.
8. Laying the foundation for further education.
9. Enabling the individual to apply acquired skills in solving problems of the community.

10. Instilling positive attitudes towards productive work and strong respect for the dignity of labour and those who engage in productive labour activities.

Aims of Teaching Subsidiary Mathematics

Teaching Subsidiary Mathematics is aimed at:

a) enabling learners acquire a range of mathematical skills that are applicable to everyday situations and other subjects they may be studying.

b) equipping learners to use mathematics as a means of communication with emphasis on clear expression.

c) inspiring learners to develop an attitude of logical thought.

d) building on the basic mathematical concepts for better understanding of the subject by the learner at A level.

e) empowering learners to construct mathematical models by:

   i) developing Mathematics to the limits of their ability.

   ii) applying it with confidence to unfamiliar real situations.

   iii) specialising in mathematical techniques required for further education or vocation.

   iv) having a positive attitude towards mathematical problem solving.

   v) appreciating the satisfaction and enjoyment that may be gained from pursuing the subject for its own sake.

   vi) presenting and interpreting mathematical information in diagrammatic, tabular and graphic form.

Target

This teaching syllabus is intended for the Subsidiary Mathematics Advanced Level Secondary school teacher. It can also be used by the learner for guidance of personal learning and practice of mathematical concepts identified in this syllabus.

Scope and Depth

The Teaching Syllabus covers Pure Mathematics, Probability and Statistics.

Teaching Sequence

The topics have been arranged in chronological order of skills acquisition. The teacher is therefore advised to follow the teaching sequence for effective teaching and learning of the subject.

Time Allocation

Subsidiary Mathematics subject is allocated 6 **periods**, each of **40** or **45 minutes** a week. This is to allow adequate time for learners to engage in a variety of learning activities and develop problem solving skills. The teacher is advised to expose learners to real world circumstances so that they are motivated to apply mathematical knowledge and reasoning in real life as much as possible. Learning by doing should be practised.

How to Use the Syllabus

This syllabus focuses on developing mathematical skills for day-to-day application and further learning. Teaching and learning of this subject should focus more on skills acquisition, development and application if the learner is to benefit from it.

The teacher should use this syllabus when he/she is developing the scheme of work or the lesson plan. The syllabus will guide the teacher to know the depth of content for each topic.

Syllabus Features

The teaching syllabus for Subsidiary Mathematics has the following features:

a) Duration

This gives the proposed number of periods for each topic, each period being 40 or 45 minutes. This is to guide the teacher to cover the syllabus adequately.

b) Learning outcome

This is the statement that specifies what the learner will be able to do upon successful completion of a given topic.

c) Competences

These define a specific range of skills, knowledge, or ability to be acquired by the learners.

The teacher should use the competences to plan the teaching and learning strategies suitable for the lesson. Competences also guide in evaluating whether learning has actually taken place.

d) Learning and Teaching Strategies

These provide the teacher with guidance on the proposed activities and (methods/methodology) that can be used in the teaching and learning. The following are the suggested learning and teaching strategies:

i) **Teacher guided research:** An instructional technique where the teacher gives students areas to read individually or in groups and later have a class discussion.

ii) **Peer presentation:** An instructional technique where the students share the knowledge they have in class with the teacher's guidance.

iii) **Brainstorming:** A technique used to gather ideas spontaneously contributed by learners. It is an effective way to generate ideas on a specific issue and then determine which idea(s) is the best.

iv) **Teacher exposition:** An instructional technique where you put the topic into context for the learner to elicit his/her contributions.

v) **Simulations:** A representation of the behaviour or characteristics of one system through the use of another system especially a computer program designed for the purpose.

The suggested teaching and learning strategies in this syllabus are not an end in themselves. The teachers are encouraged to devise other teaching strategies to enable the learners develop competences described in this syllabus.

e) Guidance to the teacher

This is to guide the teacher to prepare for lessons on a given topic. It spells out the areas of emphasis and tools to be used in a given topic.

Mode of Assessment

Continuous Assessment

This shall be carried out by the subject teacher within the provided teaching time to determine the learner's progress. It can be done in a variety of forms such as written, oral or practical, real life or abstract, completed individually or as a group.

A variety of approaches should be used so that a learner can draw inferences about learning based on information obtained through broad approaches like observing, questioning, discussing and testing.

The assessment should equally focus on all the sections/parts of the syllabus. Continuous Assessment should help the learner to:

1. apply relevant mathematical concepts, terminologies and notations.
2. recall accurately and successfully use appropriate manipulative techniques.
3. recognise the appropriate mathematical procedure for a given situation.
4. apply combinations of mathematical skills and techniques in solving problems.
5. present mathematical work, and communicate conclusions, in a clear and logical way.

Assessment Strategy

i) **Exercises in class:** These should be done in the teacher's presence and marked.

ii) **Assignments:** Learners should be given homework to enable mastery of the content, should be marked and corrections made. Remedies should be given where necessary.

iii) **Short test:** This should be at the end of the sub topic.

iv) **Summative assessment**: This shall be done at the end of the two years of Advanced Level education by Uganda National Examinations Board. This assessment shall contribute 100% of the marks the learner finally gets. The examination will be formatted as follows:

There will be one paper of **2 hours 40 minutes**. The Paper will consist of **two** sections: Section A and Section B.

Section A will comprise **Eight (8)** short compulsory questions. **Four (4)** from Pure Mathematics and **Four (4)** from Statistics only. Each question in this section will carry 5 marks giving a total of 40marks.

Section B will consist of two parts; **PURE** in part 1 and **STATISTICS** in part 2, each with four questions. Candidates will be required to answer **FOUR (4)** questions from this section, selecting at least **ONE** question from each part. Each of the four questions attempted in this Section will carry 15 marks, giving a total of 60marks.

Outline of the Teaching Syllabus

Senior Five	Term 1
Pure Mathematics	Statistics
• Matrices and their Applications • Linear Programming • Quadratics	• Statistics - Types of Data - Organisation of Data - Measures of Central Tendency - Measures of Dispersion
Senior Five	Term 2
• Indices, Logarithms, Surds • Series - Arithmetic Progression - Geometric Progression • Permutations and Combinations	• Moving Averages • Index Numbers
Senior Five	Term 3
• Vectors • Trigonometry - General Angle - Pythagorean Identities	• Scatter Diagrams and Correlation • Probability Theory
Senior Six	Term 1
• Differentiation - Derivative of a Function - Second Derivative - Curve Sketching - Maximum and Minimum - Displacement, Velocity And Acceleration	• Random Variables - Discrete Random Variables. - Binomial Distribution
Senior Six	Term 2
• Integration - Definite and Indefinite integrals - Area Under a Curve - Displacement, Velocity and Acceleration	• Continuous Random Variables • Normal Distribution
Senior Six	Term 3
• Differential Equations	

SENIOR FIVE TERM 1
Pure Mathematics

Topic 1: Matrices and their Applications

Duration: 10 Periods

Overview

A matrix is a rectangular array of numbers called elements or entries. Information can conveniently be presented as an array of rows and columns. The *order* of a matrix gives the format of how a matrix should be written; it is always in the form $m \times n$ where m is the number of rows and n is the number of columns in the matrix.

Learning Outcome

The learner should be able to carry out different operations on matrices and use them in the day-to - day activities.

Competences	Content
The learner: • states the order of a matrix. • formulates matrices of any order. • carries out the operations on matrices. • calculates the determinant and inverse of a 2×2 matrix. • uses the inverse of a matrix to solve simultaneous equations. • uses Crammer's rule to solve simultaneous equations.	• Operations on matrices • Matrix formation • Operations on matrices. • Determinant of a 2×2 matrix • Inverse of a 2×2 matrix. • Solution of simultaneous equations using matrices

Competences	Content
• arranges data in tabular form. • forms and applies matrices to solve real life problems.	and determinant (Crammer's rule). • Word problems

Teaching and Learning Strategies

i) Lead the learners to formulate matrices of any order from real life situations.

ii) Involve learners in numerous exercises on addition, subtraction and multiplication of matrices.

iii) Emphasise to the learners about the ***commutative*** property $(AB \neq BA)$ in multiplication and ***compatibility*** of matrices through various exercises.

iv) Make revision on the determinant, inverse of a 2×2 matrix, solving of simultaneous equations using matrices.

v) Give various exercises on solving simultaneous equations using the determinant (Crammer's rule).

vi) Give word problems relating to real life situations.

Guidance to the Teacher

i) Let the learners identify situations where the competences developed can be applied.

ii) Let all learners know the purpose of learning matrices.

iii) Guide (especially the Arts) students to solve simultaneous equations using matrix method.

Topic 2: Linear Programming

Duration: 10 Periods

Overview

Linear programming is a mathematical technique that deals with the optimisation of a linear function of variables known as the objective function subject to a set of linear inequalities known as constraints. The objective function may be profit, cost, production capacity, manufacturing or any other measure of effectiveness. The constraints may be imposed by different resources such as labour, finance, materials, machines, market, technology, etc.

Linearity refers to a mathematical expression in which all expressions among the variables are linear. The word "linear" means that the elements in a situation are related that they can be plotted as straight lines on a graph.

Linear programming problem has two basic parts:

i) The **objective function** which describes the primary purpose of the formulation – to maximise some returns(profits) or to minimise some costs (production cost, investment cost)

ii) The **constraint set**, which is a system of inequalities describing the restrictions (conditions) under which optimisation is to be accomplished.

Learning Outcome

The learner should be able to use a combination of inter dependent activities in view of available resources in order to maximise gains and minimise losses.

Competences	Content
The learner: • interprets the linear programming problem. • formulates all the required constraints (linear inequalities). • graphs the inequalities on the same axes and shades the unwanted regions. • writes the objective function.	• Inequalities • Graphs • Optimisation

Competences	Content
• uses the feasible region to optimise. • Applies linear programming in real life situations	

Teaching and Learning Strategies

Guide learners to:

i) understands the terminologies, less than, more than, at most and at least.

ii) formulate constraints.

iii) draw graphs and shade the unwanted region differentiating between dotted and continuous line.

iv) to be able to use the feasible region to optimise.

Guidance to the Teacher

i) Let the learner identify and understand what the constraints are.

ii) Let the learner identify the objective function.

iii) Use examples familiar to the learner's real life situations like in Business and Economics.

iv) Learners should have and use Grid papers.

Topic 3: Quadratics

Duration: 16 periods

Overview

Quadratic expressions are functions in which the highest power/degree of the variable is 2. A general quadratic expression in x is in the form $ax^2 + bx + c$ and the corresponding general quadratic equation takes on the form $ax^2 + bx + c = 0$ where a, b, c are constants and a is a non-zero integer.

The solutions (values of x) for $ax^2 + bx + c = 0$ are referred to as **roots** of the equation.

Learning Outcome

The learner should be able to apply knowledge of quadratic expressions for optimisation.

Competences	Content
The learner: • solves quadratic equations using factorisation completing squares and using a calculator. • identifies the roots of a given quadratic equation in terms of α and β. • forms quadratic equations using given roots. • identifies/finds min/max values by completing squares. • Applies quadratics in real life situations	• Methods of solving quadratic equations. • Sum and product of roots of a quadratic equation in terms of α and β. • Formation of quadratic equations • Maximum and minimum values (of the quadratic function).

Teaching and Learning Strategies

i) Guide the learners to discover the different methods of solving quadratic equations, factorisation, use of formulae, completing squares and use of scientific calculators.

ii) Use the method of completing the square to determine the maximum and minimum values of a quadratic expression.

Guidance to the Teacher

i) Formation of equations that require use of symmetrical functions is beyond the scope of this syllabus.

ii) The teacher should remind the learner of the use of the identities at O level where applicable.

$$x^2 - y^2 = (x + y)(x - y)$$

$$(x \pm y)^2 = (x^2 \pm 2xy + y^2)$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

iii) Consider situations where not all the two roots of a quadratic equation may be applicable.

iv) Applications of quadratics include, determining the trajectory of a projectile (in Physics) and expressing production functions, cost functions and utility functions (in Economics)

Topic 4: Statistics

Duration: 20 Periods

Overview

Statistics is a set of concepts, rules, and procedures that help us to:

i) **organize** numerical information in the form of tables, graphs, and charts.

ii) **understand** statistical techniques underlying decisions that affect our lives and well-being, and make informed decisions basing on information generated from processed data.

Statistics plays a vital role in every field of human activity. For example, it has an important role in determining the existing position of per capita income, unemployment, population growth rate, housing, schooling, medical facilities Geography etc… in a country. Statistics is a vital tool in fields like industry, commerce, trade, Physics, Chemistry, Economics, Mathematics, Biology, Botany, Psychology, and Astronomy. Statistical methods are used in research to collect, analyse, and formulate research findings in every field at higher institutions of learning.

Learning Outcome

The learner should be able to collect, present and analyse or interpret data using measures of central tendency and measures of dispersion to make informed decisions.

Sub-topic 1: Introduction to Data Collection

Duration: 4 Periods

Competences	Content
The learner: • collects data from the environment. • classifies the data under discrete and continuous data. • identifies discrete and continuous data with examples. • identifies ungrouped and grouped data. • groups data.	• Data collection • Discrete and continuous raw data • Types of data - Ungrouped data - Grouped data

Teaching and Learning Strategies

i) Guide learners to collect data from their communities; ages of children in different families and within the class, measurements of their height, waist, fingers, shoe size or any other variable may be considered.

ii) Guide learners to group their data.

Sub-topic 2: Organisation of Data

Duration: 6 Periods

Competences	Content
The learner: • presents data and selects a suitable way of presenting raw statistical data • uses the graph to estimate the measures of central tendency.	• Data presentation methods - Frequency tables - Histograms • Cumulative frequency graph (ogive) • Mode

Competences	Content
• uses the histogram to estimate the mode. • uses the ogive curve to estimate the median, quartiles, deciles, percentiles. • determines the interquartile range and semi – interquartile range.	• Median, quartiles, deciles, percentiles • Interquartile range and semi – interquartile range

Teaching and Learning Strategies

i) Guide learners on different forms of data presentation.

ii) Guide learners in groups or individually, to make presentations of their data making use of various charts and diagrams.

iii) Emphasis to be made about class limits and class boundaries.

iv) Equal class intervals to be used at this level.

v) Open class boundaries should be mentioned. $(20-,\ (30-,\ (40-$, or $-<20),\ -<30),$

vi) When drawing the Histogram or Ogive curve the horizontal axis should be labelled with the specific variable, e.g. Mass, Weight, height, etc...

vii) Data presentation and analysis should look at constructing histograms (with equal class widths only).

viii) A histogram is a special bar graph with the heights of the bars representing the frequencies of the groups and NO gaps between the bars.

NOTE:

- Shading of the histogram is not important.
- An ogive can be used to determine the median, quartiles, deciles and percentiles.

Sub-topic 3: Measures of Central Tendency

Duration: 6 Periods

Competences	Content
The learner calculates the mean, mode and median of: i) ungrouped data. ii) grouped data. Uses measures of central tendency	• Mean, mode and median

Teaching and Learning Strategies

i) Using practical examples, guide the learners to calculate measures of central tendency.

ii) Guide learners to compare sets of data and make meaningful conclusions on the data collected at the beginning of the topic.

iii) Guide learners to interpret the characteristics of the data basing on the measures of central tendency.

Sub-topic 4: Measures of Variation (Dispersion)

Duration: 4 Periods

COMPETENCES	CONTENT
The learner: • determines the range • calculates the variance and standard deviation. • Applies the measures of variation in real life situations	• Range • Variance and standard deviation.

Teaching and Learning Strategies

i) Guide learners to determine measures of dispersion.
ii) Guide learners to use measures of dispersion to make conclusions on sets of data.

Guidance to the Teacher

i) At this level most learners would have had some reasonable background knowledge on this topic. The methodologies used should aim at clarifying concepts already learned.
ii) You are advised to teach this topic as a project covering areas of data collection, presentation and analysis.
iii) Should discuss advantages and disadvantages of each measure of the central tendency.
iv) Teacher to guide the learners on the key terms, e.g. Range, Quartiles, Deciles, Percentiles, Interquartile range.
v) You should use the data from the learner's environment. This way you will help learners create interest in the topic/subject.
vi) Focus learners on the ability to interpret graphs and charts and the importance of central tendency values in order to make informed decisions.

SENIOR FIVE TERM 2
Topic 5: Indices, Logarithms and Surds

Duration: 15 Periods

Overview

Indices, logarithms and powers refer to the same concept thing and can be used interchangeably. Indices are helpful in simplification of expressions. Indices and logarithms help us to deal with such numbers more conveniently. Surds are irrational numbers which can be expressed as powers.

Knowledge of logarithms is:

i) commonly used in computation of numbers

ii) essential for further learning in other related subjects.

Learning Outcomes

The learner should be able to:

i) simplify and evaluate expressions involving logarithms, indices and surds.

ii) use logarithms in computations.

Sub-topic 1: Indices

Duration: 5 Periods

Competences	Content
The learner: • applies the laws of indices to carry out mathematical operations. • simplifies expressions involving all types of indices.	• Types of indices. (Positive, Negative, Fractional) • Equations reducing to quadratics

Teaching and Learning Strategies

i) Through multiplying the number by itself, guide the learners to use powers.
ii) Use the division approach to introduce negative powers.
iii) In groups or individually use the laws of indices to simplify expressions and solve equations.
iv) Solve the equations involving indices reducing to quadratic equations.
E.g. Solve the equation: $2^{2x} - 5(2^x) + 4 = 0$.

Sub-topic 2: Logarithms

Duration: 5 Periods

Competences	Content
The learner: • relates logarithms to indices. • applies laws of logarithms to mathematical expressions. • changes bases of logarithms. • simplifies logarithmic expressions in different bases. • solves linear equations involving logarithms.	• Logarithm as an index • Laws of logarithms • Change of base • Logarithmic expressions • Simple equations involving logarithms leading to linear equations.

Teaching and Learning Strategies

Through exposition, guide the learners to state and apply the laws of logarithms.

Sub-topic 3: Surds

Duration: 5 Periods

Competences	Content
The learner: • identifies rational and irrational numbers. • simplifies surds.	• Rational and irrational numbers

Competences	Content
• performs operations (adds, subtracts multiplies). • rationalises surds.	• Surds • Rationalisation of surds

Teaching and Learning Strategies

i) Through teacher exposition, define rational and irrational numbers with examples and explain the significance of rationalisation.

ii) Guide learners to rationalise the denominator.

iii) Provide activities for learners to practise and develop skills in rationalising the denominator.

iv) **DO NOT** include double surds e.g. $\sqrt{x+2}+\sqrt{x+3}=10$.

Guidance to the Teacher

i) Guide the learners to identify where this knowledge can be applied.

ii) Emphasise on group presentations so that the weak learners get more time to internalise the content and the approaches used.

Topic 6: Series

Duration: 15 Periods

Overview

A **sequence** is a set of numbers expressed in a definite order. A **series** is the sum of the terms of a sequence. The concept of series is utilized in institutions like banks, insurance companies and others. Series are classified into two i.e. the Arithmetic progression (A.P) and the Geometric progression (G.P).

An Arithmetic Progression is a sequence of numbers in which any term can be obtained from the previous one by adding a number called the *common difference, d*.

A Geometric Progression is a sequence of numbers in which any term can be obtained from the previous one by multiplying a number called the *common ratio, r*.

Learning Outcome

The learner should be able to distinguish between Arithmetic Progressions and Geometric progressions, generate both the A.P. and G.P. and applies to business related problem.

Sub-topic 1: Patterns and Sequences

Duration: 5 Periods

Competences	Content
The learner: • identifies patterns. • generates sequences. • identifies the characteristics of a sequence. • identifies the characteristics of a series. • writes a series of a function using the summation notation.	• Patterns • Sequences • Series • Series in summation notation

Teaching and Learning Strategies

i) Discusses with the learners the special notation for representing sequences and series.

ii) Teacher to guide the learners to develop patterns for logical thinking.

Sub-topic 2: Arithmetic Progression

Duration: 5 Periods

Competences	Content
The learner: • identifies A.Ps. • generates an Arithmetic Progression (A.P). • determines the n^{th} – term of an A.P by using a formula. • finds the sum of the first n terms of an A.P. • determines the value of n given the sum of an A.P and common difference d or last term l of an A.P. • applies arithmetic progression in life situation	• Arithmetic progression. • n^{th} – term of an Arithmetic Progression. • Sum of the first n terms of an Arithmetic progression.

Teaching and Learning Strategies

i) Use the sum of a set of natural numbers, sum of squares of the natural numbers to guide the learners to express them using summation notation.

ii) Prepare different sets of numbers both positives and negatives to help the learners develop an A.P.

iii) Using the above sets leads the learners to show how the formula for the n^{th} term of an A.P, $U_n = a + (n-1)d$ is derived at.

iv) Derive the formulae $S_n = \frac{n}{2}(2a + (n-1)\,d)$ and $S_n = \frac{n}{2}(a + l)$ for finding the sum of the first n terms of an A.P and guides the learners to apply them to related questions.(Learners **do not** have to derive the formula)

Sub-topic 3: Geometric Progression

Duration: 5 Periods

Competences	Content
The learner: • identifies the Geometric Progression. • identifies the characteristics of a Geometric Progression. • uses the characteristics to generate a Geometric Progression. • determines the n^{th} term of a G.P by using a formula. • uses the formula to find the sum of the first n terms of a G.P. • finds the sum to infinity using the formula. • applies G.P to real life situations.	• Geometric Progression • n^{th} term of a Geometric Progression • Sum of n terms of a Geometric Progression • Real life situations, e.g. Compound Interest, Appreciation, Depreciation, Growth, Decay

Teaching and Learning Strategies

i) Prepare different sets of numbers to guide the learners distinguish an A.P from a G.P.

ii) State the formula for the n^{th} term of a G.P, $U_n = ar^{n-1}$ and guides the learners to apply it to related problems.

iii) Apply the formulae $S_n = \frac{a(r^n - 1)}{r - 1}$ when $r > 1$ and $S_n = \frac{a(1 - r^n)}{1 - r}$ when $r < 1$ for finding the sum of the first n terms of a G.P and guides the learners to apply them to related questions.

iv) Through exposition, guides the learners to apply the formula for the sum to infinity $S_\infty = \frac{a}{1 - r}$.

Guidance to the Teacher

i) Guide the learners through this topic so that they can internalise the competences to be developed.

ii) Let the learners identify situations where the knowledge achieved is applied.

iii) Use examples from both science and Art fields.

iv) Use the mathematics tables and the list of formulae.

Topic 7: Permutations and Combinations

Duration: 14 Periods

Overview

Many situations in probability and statistics require a careful analysis of the outcomes of the events. A sequence of events occurs when one or more events follow one another. Many times one wishes to list the sequence of events, and can use several rules of counting which may include the permutation and combination rules.

We always try to arrange given objects in our homes, shops, schools and elsewhere in different ways for some reasons such as convenience and neatness. A **permutation** is an arrangement of distinct objects in a specific order. Supposing a photographer has to arrange three girls, Ann (A), Bena (B) and Halima (H) in a row for a photograph. He can do this in six possible ways: $\{A, B, H\}, \{A, H, B\}, \{B, A, H\}, \{B, H, A\}, \{H, A, B\}$ and $\{H, B, A\}$.

A **combination** is the number of ways of selecting a group of objects from a given set of objects, e.g. an A level subject combination such as HEG, PCB etc. In a combination the order of selection is not important, that is, GEH, GHE, HGE, EGH, EHG are all the same as HEG. The difference between a permutation and a combination is that in a combination the order or arrangement of the objects is not important.

Learning Outcome

The learner should be able to determine the number of permutations and combinations of a given set of objects.

Sub-topic 1: Permutations

Duration: 7 Periods

Competences	Content
The learner: • arranges items in a row, circle • identifies a permutation • relates the number of permutations to the factorial notation. • uses the permutation formula of to find the number of ways that r objects can be selected from n items.	• Arrangement of objects in a row. circle • Concept of permutation • Factorial notation $n!$ • Permutation notation ${}^{n}P_{r}$ and the formula ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

Teaching and Learning Strategies

i) Through exposition, guide the learners to arrange objects in a row.

ii) Explain the concept of permutation and how to use the formula ${}^{n}P_{r} = \frac{n!}{(n-r)!}$.

iii) In groups, guide the learners to arrange different objects in a row, then in a circle.

Sub-topic 2: Combinations

Duration: 7 Periods

Competences	Content
The learner: • identifies the different combinations. • applies the combination notation and formula to solve related problems in real life situations.	• Ways of selecting or choosing r objects from n items. • Concept of combinations. • Combination notation $\binom{n}{r}$ or ${}^{n}C_{r}$ and the formula ${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$.

Teaching and Learning Strategies

i) Discuss with the learners their subject combinations.
ii) Through exposition explains the combination notation
iii) Guide the learners with examples on how to use the formula ${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$ to compute different combinations.

Guidance to the Teacher

i) Encourage to use the objects around the classroom like books, chairs, students, boxes and playing cards to illustrate different arrangements and selections.
ii) For systematic grouping of objects, the teacher may use O-level subjects to pick the subject combinations with or without restriction.
iii) Only simple cases of permutations and combinations should be considered for example, formation of a committee from a group of members of the same or different sex.
iv) Let the Learners use calculations to find the permutations and combinations.

Topic 8: Moving Averages

Duration: 14 Periods

Overview

Moving averages of original data is one of the methods used in business and other areas of life for forecasting. Forecasting gives a business the opportunity to plan for the future as well as for several changes. It is important to have an idea about the **trend**, that is, the underlying movement of the data in consideration. For example, there could be particular points during a year when sales are lower than the previous periods. The period could be seconds, minutes, days, weeks, months, years, decades.

Learning Outcome

The learner should be able to use data to forecast the trend of events.

Competences	Content
The learner: • identifies the trend of events. • calculates odd - point or even - point moving totals and averages. • plots the original data and moving averages on the same axes. • uses the graph of the moving averages to forecast the trend of events. • joins the points to form a trend. • Applies moving average in real life situations	• The cycle of the data • Odd point and even point moving totals • Odd and even moving averages • Graphs of original data and moving averages • Trend line • Trends of events

Teaching and Learning Strategies

i) The teacher guides the learners in identifying the cycle if not stated.

ii) Learners practise how to calculate moving totals and the corresponding moving averages depending on the cycle.

iii) The teacher guides learners to plot graphs of moving averages.

Guidance to the Teacher

i) Moving averages requires a student to **extrapolate the trend line** which is vital for forecasting.

ii) Calculations of moving totals and moving averages are better done when in tabular form.

Topic 9: Index Numbers

Duration: 14 Periods

Overview

Index numbers are statistical economic indicators which provide a measure of the relative change in some variable or group of variables at a specified date when completed with some fixed period in the past.

Index numbers are also widely used by business operators to evaluate their trading positions in relation to competitors. Business operators rely on the national indices for wages, production, prices, sales, transport charges and share prices to provide simple background information against which objective decisions may be taken. For instance, they may be used to compare the present agricultural production or industrial production, price fluctuations of commodities, with those of the past years.

The simplest example of an index number is a **price relative** or **price index**. A **base year** is always chosen and this is the year on which the price changes are based.

For a price index to be realistic it should take into account the relative importance of the commodities. The method of **weighting** is used to cater for this.

Learning Outcome

The learner should be able to calculate simple and weighted price index numbers and use them to compare relative changes in a particular situation.

Competences	Content
The learner: • calculates the Price relative. • calculates the: i) Simple Price Index number ii) Simple aggregate Price Index • calculates the: i) Weighted average Price Index ii) Weighted aggregate Price Index • determines the Value Index • determines the cost of living Index. • Uses index numbers in real life situation	• Concept of Price Relatives • Concept of Price Index • Un Weighted Price Indices • Weighted Price Index i) Weighted Aggregate Price Index ii) Value Index iii) Cost of living Index

Teaching and Learning Strategies

i) $\text{Price Relative} = \text{Price Index } I = \frac{P_1}{P_o} \times 100$

ii) $\text{Simple Aggregate Price Index} = \frac{\sum P_1}{\sum P_o} \times 100$

iii) $\text{Weighted Average Price Index} = \frac{\sum \frac{P_1}{P_o} W}{\sum W} \times 100 = \frac{\sum W I}{\sum W} = \text{Value Index}$

iv) $\text{Weighted Aggregate Price Index} = \frac{\sum P_1 W_1}{\sum P_o W_o} \times 100 = \frac{\sum P_1 W}{\sum P_o W} \times 100.$

Guidance to the Teacher

The teacher should emphasize that when we are given the number, such as 120, referring to a price index, the learner must remember that the % sign is implied, that is, the learner must use 120% or 1.2 in the calculation.

The teacher is advised to use Business/ Economics textbooks.

SENIOR 5 TERM 3
Topic 10: Scatter Diagrams and Correlations

Duration: 18 Periods

Overview

Scatter diagram is a tool for analysing relationships between two variables; one variable is plotted on the horizontal axis and the other on the vertical axis. The pattern of their intersecting points can graphically show relationship patterns. Scatter diagrams are used in research to investigate relationships between two variables such as cause-and- effect relationships.

Learning Outcome

The learner will be able to interpret scatter diagrams for bivariate data and use it to draw suitable conclusions.

Competences	Content
The learner: • draws a scatter diagram and determines the nature of correlation between variables. • draws the line of best fit on a scatter diagram which MUST pass through the mean point. • determines coefficient of correlation. • draws conclusion using the coefficient of correlation.	• Concept of correlation • Scatter diagrams • Rank correlation coefficient (Spearman's) • Applications of coefficient of correlation

Teaching and Learning Strategies

i) Prepare a project for learners to investigate the correlation between two variables.
ii) Guide learners to plot a scatter diagram, draw line of best fit, and use it to make conclusions.
iii) Guide the learner to calculate the correlation coefficient using Spearman's coefficient of rank correlation (ρ_s). Comment on the result. (list of formulas is provided
iv) Guide the learner on how to comment on the value of the correlation coefficient

Guidance to the Teacher

- The teacher is advised to review straight line graphs on which most points lie. That is consider data which will form straight lines when plotted on a grid paper.
- Allow the learners to describe their understanding of scatter diagrams and the line of best fit.

Topic 11: Vectors

Duration: 18 Periods

Overview

A vector may be described as a quantity associated with a particular direction in space. Any vector may be represented by a directed line segment, whose direction is that of the vector and whose length represents its magnitude. Vectors can be added together (vector addition), subtracted (vector subtraction) and multiplied by scalars (scalar multiplication).

A vector from a point A to a point B is denoted $\vec{\mathbf{AB}}$. If coordinates are involved, we use x, y in 2 – dimensions, the unit vectors in the respective directions are $\mathbf{i}, \mathbf{j}$. Any other quantity with only magnitude is known as a scalar quantity.

Vectors are applied in various fields of science and mechanics that involve magnitude (size) and direction, for example, displacement, velocity, force and acceleration have a size or magnitude and direction. Vectors are also employed in navigation by people who operate ships and aircrafts.

Learning Outcome

The learner should be able to carry out Mathematical operations involving vectors in 2 – dimensions; compute the angle between two given vectors.

Competences	Content
The learner: • expresses a vector in a column form $\begin{pmatrix} x \\ y \end{pmatrix}$. • expresses unit vector along $x-$axis as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $y-$axis as $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. • writes a column vector in the form $a\mathbf{i}+b\mathbf{j}$. • determines the displacement vector. • identifies a position vector. • adds and subtracts vectors. • multiplies a vector by a scalar • calculates the magnitude of a vector. • identifies parallel vectors. • identifies equal vectors. • finds the dot products of two vectors. • finds the angle between two vectors. • identifies perpendicular vectors.	• Vector notations • Displacement vector • Position vector • Addition and subtraction of vectors • Multiplication by a scalar • Magnitude and direction of a vector • Parallel vectors • Equal vectors • Dot/scalar product of two vectors • Angle between vectors • Perpendicular vectors

Teaching and Learning Strategies

- Guide the learner to apply the vector notations when representing given problems.
- Prepare work on determining the displacement vector and position vector.
- Engage learners in vector addition, subtraction and scalar multiplication.

- Involve the learners with exercises to find the magnitude of a vector using the formula $|\mathbf{P}| = \sqrt{x^2 + y^2}$.
- Through exposition explains how to calculate the angle between two vectors by use of the dot product.

Guidance to the Teacher

- The teacher is advised to concentrate only on vectors in two dimensions particularly displacement vectors.
- Should make the students aware that the sum of two or more vectors is the "resultant" vector. Direction of a vector should be restricted to 2 – dimensional vectors only.

Topic 12: Trigonometry

Duration: 18 Periods

Overview

Trigonometry is a branch of mathematics that studies the relationship between the three sides and the three angles of a right angled triangle in terms of trigonometrical ratios; sine, cosine and tangent. It was developed for astronomy and geography, but scientists have been using it for centuries for other purposes too.

Knowledge of trigonometry is applicable in fields like land surveys, engineering and navigation.

Learning Outcome

The learner will be able to use trigonometry to solve problems involving circular functions and triangular shapes.

Competences	Content
The learner; • writes down the three trigonometrical ratios and their	• Expressions of the six trigonometrical ratios. The ratios of sine, cosine, tangent and their

Competences	Content
reciprocals using a right angled triangle. • obtains the trigonometrical ratios for the special angles. • deduces the sine, cosine, tangent of an angle of any magnitude using the quadrants of a unit circle. • uses Pythagoras theorem to derive trigonometrical identities. • evaluates and simplifies trigonometric expressions.	reciprocals, cosecant, secant, cotangent, respectively) • Special angles of 30^o, 45^o, 60^o • Graphs of $\sin\theta$ and $\cos\theta$ • Trigonometric identities: $\sin^2\theta+\cos^2\theta=1$ $\tan^2\theta+1=\sec^2\theta$, $\cot^2\theta+1=\cos ec^2\theta$ • Quadratic trigonometric equations No Multiple Angles, No Proofs

Teaching and Learning Strategies

i) Guide the learner to; form trigonometrical ratios from right angled triangles.
ii) Surd form of the trigonometrical ratios for special angles should be derived and simplified using appropriate triangles.
iii) Guide the learner on how to obtain all the possible angles in the given range.
iv) Guide the learner to evaluate and simplify trigonometric expressions using scientific calculators.

Guidance to the Teacher

- You are advised to revise the O-level Trigonometry to allow Learners to review the trigonometry ratios.
- Let Learners do this topic practically because this content is not totally new to them.

Topic 13: Probability Theory

Duration: 18 Periods

Overview

Probability theory is the branch of Mathematics concerned with prediction and uncertainty. It was developed from the theory of games of chance and gambling. It plays a very important role in astronomy, physics, chemistry, engineering, economics, business, social science, psychology and research.

The probability of an event is the measure of the likelihood that it will occur and it is given on a numerical scale from 0 to 1. The numbers representing probabilities can be written as percentages, fractions or decimals.

A probability of **zero** implies that the event is **impossible**. A probability of **one** (100%) indicates that the event is **certain to occur**. All other events have a probability between zero and one.

A probability event consists of one or more outcomes of a probability experiment. Two events, A and B, are said to be **mutually exclusive** if they cannot occur at the same time. Events can also be classified as **independent**. Independent events are events such that the occurrence of one does not affect the occurrence of the other.

The **complement** of an event is the set of outcomes of the event in the sample space that are not included in the outcomes of the event itself.

The **conditional probability** of an event B in relation to an event A is the probability that event B occurs after event A has already occurred.

Probability problems can be worked out by using the addition rules, the multiplication rules and the complementary event rules.

Learning Outcomes

The learner should be able to:

i) calculate probabilities from given/researched data and draw out relevant conclusions.

ii) Solve problems involving probability.

Competences	Content
The learner: • explains the terminologies and notations in probability theory. • lists down the possible outcomes of an event in an experiment. • determines probability of an event using classical probability formula. • uses Venn diagrams to solve probability problems. • determines the number of outcomes to a sequence of events using tree diagrams and desired probabilities. • determines the probabilities involving independent events using the multiplication rule. • computes the probabilities involving mutually exclusive events using the additive rule. • calculates numerical problems related to conditional probability.	• Terminologies in probability theory. • Experimental probability • Probability laws and notations in relation to set theory. • Contingency table. • Mutually exclusive and independent events. • Probability situations i.e. AND and OR. • Probability tree diagrams • The conditional probability

Teaching and Learning Strategies

The teacher should:

i) expose the learners to the knowledge about the probability theory through group work and experiments, for example, a die and a coin.

ii) guide the learners to understand the terminologies and notations in probability theory.

iii) guide the learners to state and apply the laws of probability to related problems.
iv) use the contingency table to workout probabilities.
v) use Venn diagrams to illustrate events.
vi) show how probability tree diagrams are constructed from relevant data and their application.

Guidance to the Teacher

i) Encourage the use of Venn diagrams when explaining the concept of probability.

ii) Emphasize the application of the probability laws. For example:
- $0 \leq P(A) \leq 1$
- $P(A) + P(\overline{A}) = 1$ to related problems.

iii) Picking with OR without replacement should be restricted to only 2 pickings.

SENIOR SIX TERM 1
Topic 14: Differentiation

Duration: 36 Periods

Overview

In mathematics, differential calculus is a subfield of calculus concerned with the study of the rates at which quantities change. It is one of the two traditional divisions of calculus, the other being integral calculus.

The derivative of a function at a chosen input value describes the rate of change of the function (gradient function) near that input value. The process of finding a derivative is called differentiation. Geometrically, the derivative at a point equals the slope of the tangent line to the graph of the function at that point.

Knowledge of differentiation is applicable in analysis of biology, finance and economics. One important application of differentiation is in the area of optimisation, which means finding the condition for a maximum (or minimum) to occur. This is important in business (cost reduction, profit increase), engineering (maximum strength, minimum cost) and biology (rates of muscle contraction, dissolution of drugs).

Learning Outcome

The learner should be able to obtain derivatives of functions and use them to analyse situations.

Sub-topic 1: Derivative of a Function

Duration: 9 Periods

Competences	Content
The learner: • determines the first derivative of functions. • determines the second derivative of functions	*i)* Derivatives of linear, quadratic and cubic functions $0 \leq n \leq 3$ (positive integers only) *ii)* Natural log

Competences	Content
Applies the following formula correctly in different situations $\frac{d(\ln x)}{dx} = \frac{1}{x}$	

Teaching and Learning Strategies

i) Through teacher exposition, guide learners to use the notations $\frac{dy}{dx}$ and $f'(x)$. If $f'(x)$ or $\frac{dy}{dx}$ is the first derivative of the function then $f''(x)$ or $\frac{d^2y}{dx^2}$ is the second derivative of that function.

NOTE: $\left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx} \times \frac{dy}{dx}$ which is not equal to $\frac{d^2y}{dx^2}$

ii) Take learners through the process of determining derivatives of functions.

iii) Allow learners time to practice how to determine derivatives of functions.

iv) Teachers should NOT give the proof to $\frac{d(\ln x)}{dx} = \frac{1}{x}$

v) Teachers should give simple examples that involve $\ln x$ e.g. $\frac{d}{dx}(x^2 + \ln x)$

Sub-topic 2: Maximisation and Minimisation

Duration: 9 Periods

Competences	Content
The learner: • determines the value(s) of x for which the first derivative is 0 i.e. $\frac{dy}{dx} = 0$. • identifies the values of x for which the function is either minimum or maximum. • determines either the maximum or minimum value of the function. • formulates functions from real life situations in order to minimise or maximise function.	*iii)* First and second derivative. *iv)* Formulation of functions. *e.g., Profit functions, Cost reduction functions...*

Teaching and Learning Strategies

i) Guide learner to use the second derivation in identifying maximum and minimum.

ii) Guide learners in the forming of function from word problems in real life situations.

NOTE: Do not mention points of inflexion.

Sub-topic 3: Curve Sketching

Duration: 9 Periods

Competences	Content
The learner: • finds turning points of the quadratic curve by differentiation.	• Turning points • Gradient method • Second derivative method

• determines the nature of the turning points (Maximum or Minimum points). • determines the intercepts. • sketches the quadratic curves.	• Maximum or minimum points • Intercepts • Curve sketching

Teaching and Learning Strategies

i) Prepare work on functions to be differentiated by the learners and find the root for which the first derivative is zero.
ii) Through exposition, guide the learners on how to use the signs of the gradients of the tangents.
iii) Through guided discussion, the learners find out the nature of the turning point using the sign change method.
iv) Through exposition guide the learners to find the point(s) where the curve cuts the axes.
v) Through guided discovery, help the learners to sketch quadratic curves only.

Sub-topic 4: Displacement, Velocity and Acceleration

Duration: 9 Periods

Competences	Content
The learner: • differentiates displacement function to obtain velocity function. • differentiates velocity function to obtain acceleration function.	• Displacement $s(t)$ and velocity $v(t)$ • Velocity $v(t)$ and acceleration $a(t)$

Teaching and Learning Strategies

i) Guides the learner through examples, exercises and assignments to conceptualise the relation between displacement, velocity and acceleration.
ii) Guide the learner to use the rule of differentiation.

iii) Through exposition the teacher guides the learners to differentiate velocity function to obtain acceleration function.

Guidance to the Teacher

i) Use other variables like change in profit with change in number of employees to allow learners develop their own understanding of a derivative of a function.

ii) For a function $y = ax^n$, the formula $\frac{dy}{dx} = nax^{n-1}$ can be used to guide learners obtain derivatives of simple algebraic functions e.g. $y = x^2$, $y = (x-5)(3x+7)$ $y = (2x+5)^2$, $y = 3x^3 + 2x^2 + x + 2$, etc.

iii) The variables x and y may be replaced by any other relevant variables.

iv) Differentiation from **first principles**, **product rule** and **quotient rule** should not be taught.

v) Remind the students about the general shape of the graph in the form $ax^2 + bx + c = 0$ for $a > 0$ having a minimum turning point while $a < 0$ the curve has a maximum turning point.

Topic 15: Discrete Random Variables

Duration: 36 Periods

Overview

Decision making in many businesses, insurance companies, and other real-life situations is made possible by assigning probabilities to all possible outcomes to the situations and then evaluating the results. For example, a shopkeeper can compute the probability that he will make either 0, 1, 2, 3, 4, or more sales in a single day and will be able to compute the average number of sales he makes per day, per week, which will enable him to make better predictions over a period of time say, monthly.

When carrying out an experiment, *variables* are used to describe the event.

A *variable* in this case can be defined as a characteristic that can assume different values. Letters of the alphabet such as X, Y, or Z can be used to represent variables. Since the variables are associated with probability, they are called *random variables*.

Random variables may be either *discrete* or *continuous*. A *discrete random variable* is the variable that has values that can be counted. For example, if a die is thrown, a letter such as X can be used to represent the possible outcomes, i.e. X is assigned values 1, 2, 3, 4, 5, or 6 corresponding to the outcomes. The relationship between the possible values of a random variable and the corresponding probabilities is term as the *probability distribution* of the random variable which may be specified in terms of a probability distribution function (pdf).

Sub-topic 1: Discrete Random Variables

Duration: 18 Periods

Learning Outcome

The learner should be able to identify the characteristics of a discrete random variable, compute its probabilities and other statistical measures.

Competences	Content
The learner: • identifies the: i) random variable. ii) discrete random variable. • identifies the properties of a pmf of a discrete random variable. • uses the properties of a pmf of a discrete random variable. • determines probabilities of events. • determines the mode and median of a discrete random variable. • generates and constructs a probability distribution table.	• Concept of discrete random variable. • Probability mass function (pmf) of a discrete random variable. • Properties of a pmf of a discrete random variable. • Probability distribution table. • Graph of a pmf. • Mode, median • Expectation $E(X)$, Variance, $Var(X)$, and

Competences	Content
• calculates the expectation $E(x)$, Variance, $Var(x)$, and Standard deviation of a discrete random variable.	Standard deviation of a discrete random variable.

Teaching and Learning Strategies

i) Learners throw dice and list the possible outcomes.

ii) Relate the possible outcomes with a random variable.

iii) Define a discrete random variable.

iv) The probability function is referred to as the Probability mass function (p.m.f).

v) Through exposition guides the learner to generate a probability distribution for throwing a die.

vi) Assign the learners to construct the probability distribution for a discrete random variable, "the number of heads obtained from tossing two coins "Engages the learners in tasks to find the sum of the probabilities of the distribution."

vii) Emphasize the two properties of the probability distribution of a discrete random variable.

viii) Lead the learners to represent the probability distribution in a columnar table.

ix) Involve the learners in calculating the expectation $E(X)$, variance, $Var(X)$, and standard deviation using the formulae.

x) Help the learners to familiarise with the use of the table of distribution of probability while calculating the $E(X)$, $Var(X)$ and standard deviation using various exercises and assignments.

Sub-topic 2: Binomial Distribution

Duration: 18 Periods

Overview

There are some probability situations that may result into only two possible outcomes. Such situations may include:

i) when a baby is born, it may be either male or female.

ii) when fair coin is tossed once, a head or tail are obtained.

Other situations that are reduced to only two possible outcomes may include:

i) a person taking a Pioneer bus may arrive either on time or not on time

ii) a company producing items that are either defective or not defective.

All the above mentioned situations are called binomial or Bernoulli experiments. The outcomes of a binomial experiment are classified as *successes* or *failures* denoted by p and q respectively.

Therefore, a ***binomial distribution*** is one that represents the outcomes of a binomial experiment and their corresponding probabilities.

Learning Outcome

The learner should be able to interpret a binomial distribution and use it to determine the probability of events, mean, variance and standard deviation.

Competences	Content
The learner: • identifies the characteristics of a binomial distribution. • interprets the notation $B(n,\ p)$. • calculates the probability of event using formulae, tables or calculators. • determines the $E(x)$ and $Var(x)$ of binomial distribution.	• Binomial distribution • Properties of a binomial distribution • Binomial probability formula; $P(X=r)={}^{n}C_{r}p^{r}\ q^{n-r}$ • Binomial tables • Expectation, variance and standard deviation.

Competences	Content
• applies properties of a binomial distribution.	

Teaching and Learning Strategies

i) Introduce a binomial distribution as an example of a discrete random variable.

ii) Lead the learners to identify probability situations that may result in two outcomes of success or failure.

iii) Guide the learners through the properties of a binomial experiment.

iv) State the binomial distribution and its notation.

v) Guide the learners on how to use the formula, mathematical tables and calculators to compute binomial probabilities.

vi) Involve the learners in calculating probabilities of events, expectation $E(x)$, variance, $Var(x)$, and standard deviation of a binomial distribution.

Guidance to the Teacher

i) Let the learners understand the relationship of all parameters learnt in this topic.

ii) Give examples of situations where this topic is applied. Use examples which are drawn from both Arts and Science fields.

SENIOR SIX TERM 2
Topic 16: Integration

Duration: 24 Periods

Overview

Integration is the process of obtaining an original function from a given gradient function; hence, it is the reverse of differentiation. Thus, if the rate of variation of a function is known, integration process can enable us to get the function itself. Integration is used to compute such things as the areas of irregular shapes.

$\int f(x)\,dx$ is referred to as an **indefinite integral** because it does not give a definite answer and we add an arbitrary constant after integrating.

$\int_a^b f(x)\,dx$ is referred to as a **definite integral** because it gives a definite answer, where a is the **lower limit** of the integral and b is the **upper limit** of the integral.

The definite integral can be used in a number of applications in all science related disciplines such as calculating work done, business and economics, including price discrimination, revenue versus cost, consumer's surplus and producer's surplus.

Learning Outcome

The learner should be able to integrate gradient function, evaluate definite integrals and apply the knowledge to real life situation.

Sub-topic 1: Definite and Indefinite Integrals

Duration: 8 Periods

Competences	Content
The learner: • determines indefinite and definite integrals with the constant of integration. • evaluates definite integrals. • Applies the following formula correctly in different situations $\int \frac{1}{x} = \ln x$.	• Indefinite integrals • Definite integrals • Natural logarithm

Teaching and Learning Strategies

iv) Guides the learner through examples, exercises and assignments to conceptualise the basics of integration.

v) Guide the learner to use the rule of Integration.

vi) Through exposition the teacher guides the learners to evaluate definite integrals. NOTE: Use Integrals with powers $n \geq 0$.

vii) Guide the learner to distinguish between indefinite and definite integrals.

viii) Teachers should **NOT** give the proof to $\int \frac{1}{x} dx = \ln x + C$

ix) Teachers should give simple examples that involve $\int \frac{1}{x} dx$ e.g. $\int x^2 + \frac{1}{x} \; dx$

Sub-topic 2: Area under a Curve

Duration: 8 Periods

Competences	Content
The learner: • sketches quadratic curves. • uses integration to find the area between the given curve and the $x-$ axis.	• Area under a curve

Teaching and Learning Strategy

Through various exercises guide the learner to find the area under a curve

Guidance to the Teacher

The teacher should:

i) emphasise the Integration sign as a long S, and also indicate that the integration is with respect to a variable say x.

ii) emphasise the constant of Integration with indefinite integrals. We know that $y=x^3$, $y=x^3+5$, $y=x^3-6$, all satisfy $\frac{dy}{dx}=3x^2$, for this reason we write $y=x^3+c$ after integrating because we do not know whether the original function had a constant term or not.

iii) remind the learners that when working with definite integrals, the constants of integration are excluded in the final result.

iv) emphasise sketching the quadratic curves to show clearly the area covered.

Sub-topic 3: Displacement, Velocity and Acceleration

Duration: 8 Periods

Competences	Content
The learner: • integrates acceleration function to determine the velocity function. • integrates the velocity function to determine the displacement function.	• Velocity • Acceleration

Teaching and Learning Strategies

i) Involve the learner to link the terms displacement, velocity and acceleration to the process of differentiation w.r.t. time i.e. $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$.

ii) Lead the learners to link together the terminologies acceleration (a), velocity (v) and displacement (s) to the process of integration i.e. $s = \int v\,dt$, $v = \int a\,dt$ for integral powers $n \geq 1$.

Guidance to the Teacher

i) Let the learner explain the relationship between displacement, velocity and acceleration with respect to integration.

ii) Explain to learners how differentiation and Integration are used in this topic.

iii) Guide the learner to identify where the knowledge achieved in this topic is applied.

Topic 17: Continuous Random Variables

Duration: 18 Periods

Overview

A *continuous random variable* is the variable that takes on uncountable domain e.g. height, weight, distance. The continuous random variable takes on values in the interval $a < x < b$ or (a, b).

The relationship between the possible range of values of a random variable and the corresponding probabilities is termed as the *probability distribution* of the continuous random variable which may be specified in terms of a probability density function (pdf).

Learning Outcome

The learner should be able to identify the characteristics of a continuous random variable, compute probabilities, mean, variance and standard deviation.

Competences	Content
The learner: • identifies the characteristics of a continuous random variable. • computes the probabilities, expectation, variance and standard deviation of a continuous random variable in order to analyse data.	• Concept of continuous random variable. • Properties of a Continuous random variable • Probability density function • Expectation, variance and standard deviation of a continuous random variable.

Teaching and Learning Strategies

i) Describe a continuous random variable.

ii) Introduce to the learners the p.d.f of a continuous random variable as an area under the graph of the given function. I.e. $P(a \leq X \leq b) = \int_a^b f(x)\,dx$.

iii) Emphasize the properties of the probability distribution of a continuous random variable.

iv) Involve the learners with various exercises and assignments in calculating the probabilities, expectation $E(X)$, variance, $Var(X)$, and standard deviation using the formulae.

Guidance to the Teacher

i) Guide the learner to differentiate discrete and continuous random variables.

ii) Let the learners discuss this topic and come up with situations where the competences in the topic can be applied.

Topic 18: Normal Distribution

Duration: 18 Periods

Overview

The normal distribution is a continuous, bell-shaped distribution of a variable symmetrical about the mean. It is the most important continuous probability distribution for both practical and theoretical statistics. The normal distribution provides a good probability model for many continuous variables whose values depend on the effect of a number of factors; such variables may include the age, heights of people, weights and other measurements academic performance of learners.

Learning Outcome

The learner should be able to use characteristics/properties of a normal distribution to determine probabilities for a normally distributed variable in order to make informed decisions.

Competences	Content
The learner: • identifies the properties of the normal distribution. • interprets the notation $N(\mu, \delta^2)$ of normal distribution. • standardises the random variable into the standard normal variable Z . • determines the area under the standard normal curve given various z values. • uses the standard normal distribution tables or calculator to determine probabilities. • determines mean and standard deviation from given probabilities.	• Concept of normal distribution • Properties/characteristics of normal distribution • Standardization • Standard normal distribution tables • Calculators

Teaching and Learning Strategies

i) Introduce a normal distribution as a continuous random variable.

ii) Guide the learners to outline the properties of the normal distribution.

iii) Guide the learners to:

a) Sketch the normal curve.

b) Shade the desired area.

c) Find the probability for the corresponding z value in the table.

d) Find z value corresponding to a given probability

iv) Lead the learners to use the formula for a standard score to transform the normally distributed variable into the standard normal variable.

v) Guide the learners to use the mathematical tables for cumulative normal distribution through various exercises and assignments.

vi) Assign the learners in groups to determine the probabilities for a normally distributed variable by transforming it into a standard normal variable.

Guidance to the Teacher

Guide learners to identify areas in real life situations where normal distribution is applied.

SENIOR SIX TERM 3

Topic 19: Differential Equations

Duration: 30 Periods

Overview

We learnt that the derivative is also an instantaneous rate of change i.e. we denoted the instantaneous rate of change of distance y with respect to time t as $\frac{dy}{dt}$. For many growth processes, the rate of change of the amount of a substance with respect to time is proportional to the amount present. This can be represented by the equation $\frac{dy}{dt} = ky$ where k is a constant.

An equation of this type, where y is an unknown function of say x, is called a **differential equation** because it contains derivatives or differential coefficients. There are several methods of solving differential equations. However, in our case we are **only** going to use the method of **separation of variables** to solve these equations.

Differential equations are used to solve applied problems such as those involving carbon dating and radioactive decay, the amount of drug in an organ, mixtures, supply and demand, logistic growth, economic profit and marginal productivity.

Learning Outcome

The learner should be able to use integration to find the general solution of a differential equation, and find particular solutions of differential equations in initial value problems.

Competences	Content
The learner: • identifies a differential equation	• Concept of a differential equation.

Competences	Content
• finds the general solution and particular solution of a differential equation. • solves problems involving separable differential equations related to natural occurrences.	• General solution and particular solution of a differential equation. • First order differential equations with separable variables. • Differential equations of natural occurrences.

Teaching and Learning Strategies

i) Introduce using equations such as $y = x^2$ and involve them to find the first derivative.

ii) Relate the above derivatives to the definition of a differential equation.

iii) Define the order of a differential equation and prepares work for the learners to identify the different orders.

iv) Engage the learners in several tasks to use integration to determine the general and particular solution of first order differential equations.

v) Prepare differential equations with separable variables and guides the learners to separate and integrate them.

vi) Provide word problems with separable variables for the learners to solve the differential equations.

Guidance to the Teacher

Let the learners identify where the competences achieved/ developed are applied in real life situations.

Bibliography

Backhouse, J. (2011). *Pure Mathematics.* International. Harlow: Pearson Education Limited.

Chandler, L., & S., B. (1996). *Mechanics and Statistics for Advanced Level.*

Crawshaw, J., & Chambers, J. (2015). *Advanced Level Statistics.*

Smedley, R., & Wiseman., G. (1998). *Introducing Pure Mathematics.* Oxford University Press.

Thorning, A., & Sadler, D. (1996). *Understanding Pure Mathematics.* United Kingdom: Oxford University Press.

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